A small tour of optimization models

Theory of games and markets with examples

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OPTIMIZATION AND APPLICATIONS

Consulting:
jonas2@optimum2.mii.lt
Web-sites:
$http://soften.ktu.lt/\tilde{}mockus$
$http://pilis.if.ktu.lt/\tilde{}jmockus$
$http://eta.ktl.mii.lt/\ mockus$
$http://mockus.us/optimum$ (short)

Optimality

Objective function

$$f(x),\ x = (x_1, ..., x_m). \tag{1}$$

Global minimum

$$f(x_A) \leq f(x)\ x \in A, \tag{2}$$

or

$$x_A = \arg\min_A f(x). \tag{3}$$

Here A is a feasible region.

Local minimum

$$f(x_\epsilon) \leq f(x)\ x \in \epsilon. \tag{4}$$

Here ϵ is a vicinity of x_ϵ.

Vector optimization

Objective is to maximize vector-function

$$f(x) = (f_i(x),\ i = 1, ..., m). \tag{5}$$

Pareto optimum is the set X^*.
$x^* \in X^*$, if there are no such x that

$$f_i(x) \geq f_i(x^*),\ \forall i \tag{6}$$

$$f_j(x) > f_j(x^*),\ \exists j \tag{7}$$

Scalarization

By weights

$$x(c) = arg\max_x \sum_i c_i f_i(x),\ c_i > 0. \tag{8}$$

Here $x(c) \in X^*$.

By constraints

$$x(b) = arg\max_x f_1(x) \tag{9}$$

$$f_i(x) \geq b_i,\ i = 2, ..., m. \tag{10}$$

Here $x(b) \in X^*$, if $x(b)$ is unique,
otherwise, non-Pareto points are possible.